

Modelling of Traffic System with Time-Variant Saturation Flow

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Abstract— This paper deals with a new approach for modelling of time-variant saturation flow and turning movements. Generally, these traffic quantities are considered as time-invariant in modelling. It emerged that neglecting of time-variant properties of the model quantities can cause significant worsening of the model quality. The new approach determines the value of actual saturation flow on the basis of measurements on two successive detectors. The knowledge of the actual saturation flow allows to determine the actual turning movement as well. The goal of the paper is twofold. First, to describe the traffic model with stress laid on the modelling of the time-variant saturation flow and relation between the saturation flow and turning movements. Second, to use the nonlinear filtering method for estimation of unmeasurable quantities (queue lengths) in the nonlinear traffic model with and without time-variant saturation flow.

I. INTRODUCTION

Traffic congestion is very affected by setting of traffic lights (traffic light controllers) in cities. The driver loses time and the car uses fuel when waiting for traffic lights. For that reason, various dynamic traffic control methods have been deployed [1], [2]. These methods are usually not easily adaptable to the dynamic control of historical urban areas.

For cities with historical centres, such as Prague, the capital of the Czech Republic, the dynamic traffic control strategy is being designed [3], [4], [5]. The designed traffic model is based on the traffic flow conservation principle, which allows to design the traffic control law minimising sum of queue lengths of waiting vehicles in the traffic network.

The traffic system behaviour can be generally characterised by the time-variant quantities¹ describing the traffic flow (intensity, occupancy, queue length, etc.), the time-variant quantities describing an intersection layout (turning

movements, saturation flow, etc.), and the non-physical time-variant traffic parameters. To simplify the model of the traffic model, many of the time variant quantities are, in different traffic control strategies, often modelled as time-invariant [1]. However, this simplification can causes the significant worsening of the traffic model quality which leads to degradation of used estimation or control methods. Therefore, the main aim of the paper is to model the time behaviour of the saturation flow and to find a relation between the saturation flow and turning movements. The subsequent aim is to use the nonlinear local filtering method for estimation of unmeasurable quantities (queue lengths) in the nonlinear traffic model with and without time-variant saturation flow and to compare the results.

The paper is organised as follows. Section II provides a description of the traffic quantities and introduces the traffic model design technique. The novel relation modelling the time behaviour of the saturation flow and the relation between the saturation flow and turning movements are given in Section III. In Section IV the state estimation method used for the estimation of directly immeasurable queue length is briefly described. Finally, the example and concluding remarks are given in Section V and VI, respectively.

II. TRAFFIC MODEL

In this section, the basic description of the traffic system model will be introduced. The traffic model is constructed with respect to the traffic system and its parameters and quantities.

A. Traffic quantities

Quantities that characterize the traffic system can be divided into two groups:

(i) Quantities determined by the *intersection layout and configuration*

- **Saturation flow** S_k corresponds to the actual number of cars flowing through the intersection lanes per hour - given in $[uv/h]$, where uv represents a unit vehicle. This quantity mainly depends on the road width, number of traffic lanes in one direction, and the turning movement.
- **Turning movement** $\alpha_{i,j}$ is the ratio of cars going from the i -th arm to the j -th arm [%].
- **Cycle time** t_c is a period of a phase change of the traffic light [s]. Mention that, the cycle time is assumed to be time-invariant and equal to the measurement period in this paper.
- **Green time ratio** z_k is the ratio of the effective green time to the measurement time period [%]. Note that the green time ratio is usually defined as a ratio of the

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¹Note that some of the quantities are not directly measurable by inductive detectors which represent the very common measurement device of a traffic flow in cities. The inductive detector is a inductive loop built in a cover of road, which is activated by a passing vehicle. The road (or lane) can be equipped by one detector on the output and three detectors on the input: (i) detector on the stop line, (ii) outlying detector (30 m before the stop line), (iii) strategic detector - (100 m before the stop line). Ideally, each lane has all three types of the detectors but in real traffic system, the lane is usually equipped by one or two types of such detector, due to the constrained conditions e.g. in the old urban areas with short lanes. The strategic detectors, which are most remote from a stop line, give the best information about the traffic flow at present. The detectors are able to measure the intensity, occupancy, etc.

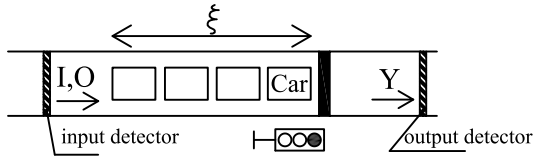


Fig. 1. An example of a one-way road.

effective green time to the cycle time. However, such definition can lead to the possible discrepancy between the cycle time and measurement period which can lead to significant problems.

- **Offset** is the difference between the start (or end) of green times at the two adjacent signalized intersection [s].

(ii) Quantities describing the *traffic flow*

- **Input intensity** \mathcal{I}_k or **output intensity** \mathcal{Y}_k (at time instant k) is the amount of passing unit vehicles per hour $[uv/h]$ measured by the detector placed in the input or output lane, respectively.
- **Occupancy** O_k is the proportion of the period when the detector is occupied by vehicles [%].
- **Queue length** ξ_k is a maximal number of vehicles waiting in one lane at a measurement period $[uv/period]^2$.

Although, majority of quantities in the previous list is assumed as units per hour, in the designed traffic model all these quantities are recalculated with respect to the respective time period of the measurement. It means, that saturation flow S_k is given in unit vehicle per hour $([uv/h])$, but in the model the corresponding recalculated quantity S_k is given in unit vehicle per period $([uv/period])$. The same goes for the input intensity \mathcal{I}_k and the output intensity \mathcal{Y}_k . As the model is discrete then the dependence on period can be omitted, i.e. the quantities are given only in their respective units.

B. Traffic state-space model

For the sake of simplicity, the state-space model will be introduced for a simple one-arm micro-region depicted in figure 1.

The state x_k is formed by the queue length ξ_k , input intensity I_k , and occupancy O_k , where the last two are measured on input strategic detector. The measurement y_k is formed by the input intensity I_k and occupancy O_k measured on strategic detector, output intensity Y_k measured on output detector and intensity J_k measured on stop-line. The quantities which are assumed as the system input u_k are green time z_k , cycle time t_c and intensity of oncoming vehicles \bar{J}_k [6].

The state-space model for one-arm intersection, see figure 1, is given as

²The queue length can be also considered in meters. The length of unit vehicle is supposed to be 6 m.

$$\begin{aligned}
 x_{k+1} &= \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \begin{bmatrix} \xi_{k+1} \\ I_{k+1} \\ O_{k+1} \end{bmatrix} = \\
 &= \begin{bmatrix} \xi_k + I_k - f(\xi_k, I_k, V_k) + w_{1,k} \\ I_k + w_{2,k} \\ \kappa_k \xi_k + \beta_k O_k + \lambda_k + w_{3,k} \end{bmatrix} \quad (1) \\
 y_k &= \begin{bmatrix} I_k \\ O_k \\ Y_k \\ J_k \end{bmatrix} = \begin{bmatrix} x_{2,k} + v_{1,k} \\ x_{3,k} + v_{2,k} \\ f(x_{1,k}, x_{2,k}, V_k) + v_{3,k} \\ f(x_{1,k}, x_{2,k}, V_k) + v_{4,k} \end{bmatrix} \quad (2)
 \end{aligned}$$

where $f(\cdot)$ is the nonlinear function characterising the number of outgoing vehicles, \bar{J}_k is measured number of passing vehicles in uv , V_k is maximal number of vehicles which are able to passing through intersection in time k (maximal capacity). The parameters β_t and κ_t reflect the inherent relations between state variables and parameter λ_t is a correction term to omit a zero occupancy, because the zero queue length and occupancy at the previous time instant do not imply the zero occupancy at the actual time instant. The variables $\mathbf{w}_k = [w_{1,k}, w_{2,k}, w_{3,k}]^T$ and $\mathbf{v}_k = [v_{1,k}, v_{2,k}, v_{3,k}, v_{4,k}]^T$ represent the state and measurement noise, respectively, which are supposed to be mutually independent and independent of the initial state of the system.

The function describing the number of outgoing vehicles $f(\cdot)$ should be equal to the number of the passing vehicles \bar{J}_k . Note that for this simple micro-region, also the intensities J_k and Y_k are the same. The function $f(\cdot)$ depends on the previous queue ξ_k , input intensity I_k , and actual value of the maximal capacity V_k . The actual value V_k is depended on the actual saturation flow S_k , green time z_k and measurement period T .

The quantity V_k can be modeled as

$$V_k = S_k \cdot \frac{z_k}{T} \quad (3)$$

where T is measurement period in s .

The function $f(\cdot)$ describes the relation between the number of vehicles which want to pass through an intersection (i.e. number of cars in a queue plus arrived cars) and the number of passing vehicles, which depends on the maximal capacity of the intersection. The function can be expressed as

$$f(\xi_k, I_k, V_k) = V_k \left(1 - e^{-\frac{1}{V_k}(\xi_k + I_k)} \right) \quad (4)$$

The regular description and derivation of function $f(\cdot)$ can be found e.g. in [4].

The remaining quantities, namely the actual saturation flow and its influences on capacity of road (lane), is described in more detail in the following section.

It should be also mentioned that example of modelling of more complex micro-regions can be found e.g. in [4], [5].

III. SATURATION FLOW AND TURNING MOVEMENTS

The time-invariant saturation flow can be used in cases when one way road with protected turning movement is

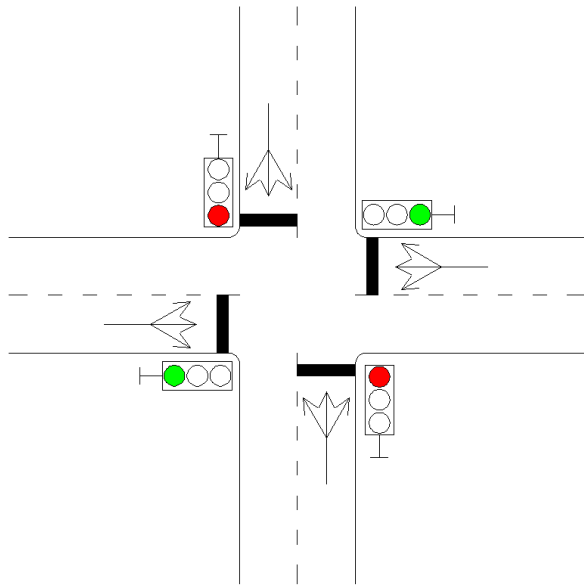


Fig. 2. Two-way four-arm intersection, where the vehicles can turn into three adjacent arms.

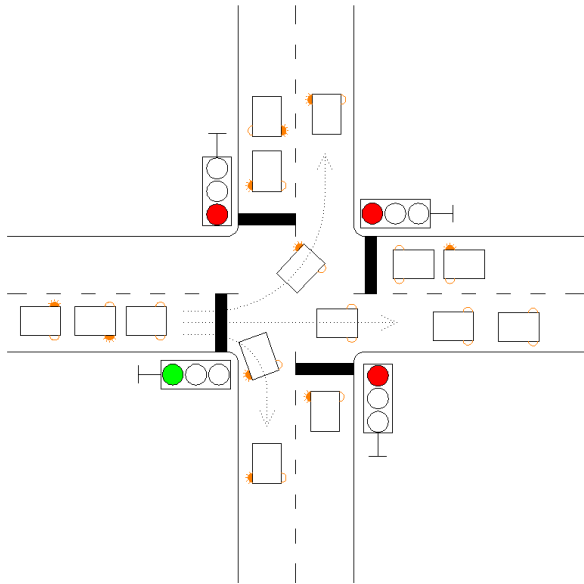


Fig. 3. The four-arm intersection with protected left turn lane.

assumed, such as highways. In other cases one value of the time-invariant saturation flow for whole day can cause the wrong value for calculation of the maximal capacity (3). Actual saturation flow is especially depended on turning movements and intensities.

Let one two-way four-arm intersection, where the vehicles can turn into three adjacent arms, be considered (see figure 2). In figure 3, it is seen the case when the traffic flow is protected. The saturation is different in cases when whole traffic flow continues straight and when the cars go to the three separate direction (turn left, turn right, and straight). The difference is caused by the different speeds for safety passing through intersection. In figure 4, it is seen the case

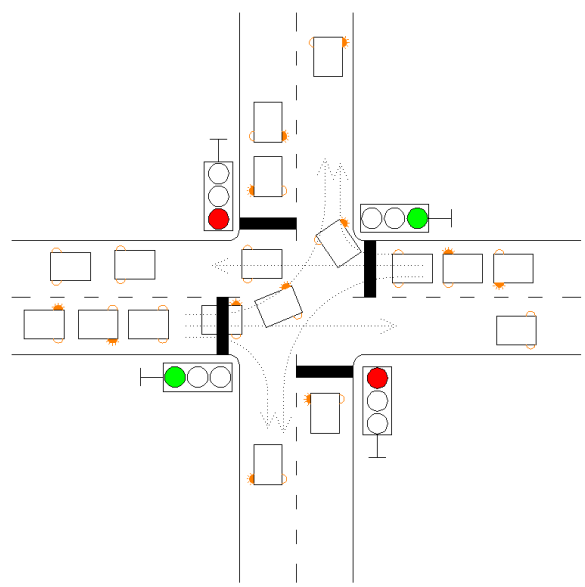


Fig. 4. The four-arm intersection with permitted left turn lanes.

when the traffic flow is permitted. The saturation flow is different from the previous case because the left movement is influenced by the oncoming vehicles and by the pedestrian crossings in a conflict crosswalk.

As was discussed in the previous text, the usage of the modelled time-invariant saturation flow in real traffic control is not suitable and it can lead to the significant errors in estimation of immeasurable quantities or in control of the traffic flow. From that reason, it is possible to replace the time-invariant saturation flow by the predefined values or function of the saturation flow.

The first approach supposed the predefined values. The traffic situation will be evaluated off-line and for selected hours or selected combination of intensities and turning movements will be assigned the value of the saturation flow.

The second approach describes the saturation flow as a time-variant function. Actual saturation flow is depended on several quantities. The saturation flow [6] is standardly described by a piecewise constant function, which is not suitable in many situations. Thus, in this paper it is supposed that the actual saturation flow is described by the following relation

$$S_k = S_0 \gamma_k - \alpha_k^R c_{1,k} \gamma_k - \alpha_k^L (c_{2,k} - c_{3,k} \bar{J}_k) \gamma_k, \quad (5)$$

where S_0 is theoretical and maximal saturation flow in uv , $\alpha_k^{R(L)}$ is right (left) turning movement in %, γ_k is factor for heavy vehicles, $c_{1,k}$ is coefficient for the right turning movement which is determined by radius of right turn, $c_{2,k}$ is coefficient for the left turning movement which is determined by radius of left turn, $c_{3,k}$ is oncoming vehicles coefficient. \bar{J}_k is the oncoming intensity, i.e. the intensity of oncoming vehicles measured by the stop-line detector.

The factor for heavy vehicles γ is time-variant but it is measurable in special cases only. The distinguishing of heavy and light vehicles can be made on places, where

the road is equipped by vehicle weighing machine. The weighing machines are placed very rarely in the city and for that reason, the factor for heavy vehicle γ will be mostly supposed to be time-invariant. The γ will be set as average value over whole day.

Parameters $c_{1,k}$, $c_{2,k}$, and $c_{3,k}$ are estimated by suitable estimation methods as it is discussed in the following section.

1) *Oncoming vehicles*: The number of oncoming vehicles can be obtained from model or directly measured. The measurement can be used only in case when the road is equipped by the stop-line detector. For traffic control we need to make prediction of oncoming intensities. On the other hand, it can be assumed that the intensity on stop-line is relatively uniform and so this intensity can be written as

$$\overleftarrow{J}_k = \overleftarrow{J}_{k-1} + e_k \quad (6)$$

where e_k is random variable. The intensity of oncoming vehicles can be influenced by green signal. It means that number of oncoming vehicles can be decreased by short green time.

2) *Time-variant turning movements*: The actual values of turning movements are given by traffic flow structure. This quantity is not measurable and with respect to the abrupt changes in behaviour of traffic flow it is not possible to estimate the actual turning movements.

In different time periods, the turning movements are different and with different impact on the traffic flow. It means that the influence on saturation flow and consequently on maximal capacity is different depending on the actual traffic situation. Strong left movements has big bearing on saturation flow in case that is permitted and the oncoming intensity is significant. In case, that oncoming intensity is small or the left movements are protected, the saturation flow is not rapidly reduced.

In equation (5), it is seen that the turning movements $\alpha^{R(L)}$ are time-invariant but there are „turning movements” parameters $c_{1,k}$, $c_{2,k}$, and $c_{3,k}$. These parameters are time-variant and they are estimated with respect to the actual value of the saturation flow. In such case, that the lane is equipped by stop-line detector are rectified according to the real value of measured saturation flow. This technique leads to possibility to compute the relatively accurate value of the left or right turning movements.

IV. USED ESTIMATION METHODS

Application of the state estimation techniques is conditioned by the knowledge of the sufficiently exact model. However, the model presented in previous section contains several unknown parameters, in both “deterministic” and “stochastic” part, which cannot be determined from the physical properties of the traffic system, namely parameters β_k , κ_k , λ_k , $c_{1,k}$, $c_{2,k}$, $c_{3,k}$, and the statistics of the state and measurement noises. Therefore, the unknown parameters have to be identified somehow.

Generally, there are two possibilities how to estimate the state and the parameters in the deterministic part of

the system. The first possibility is based on an off-line identification of the unknown parameters, e.g. by prediction error methods [7], and subsequently on an on-line estimation of the state by a suitable filtering algorithm [8]. However, off-line identified time-variant or invariant parameters represent average values rather than the actual (true) parameters and this approach is therefore suitable for traffic systems where intensity of the traffic flow is almost constant. The second possibility is based on the simultaneous on-line estimation of the state and the parameters by extension of the state with vector of the unknown parameters [9], [10]. This leads to the nonlinear model of the traffic system and to the application of the suitable nonlinear estimation method. There are two main groups of nonlinear estimation methods, namely local and global methods. Although, the global methods are more sophisticated than local methods, they have significantly higher computation demands. Due to the computational efficiency, the stress will be mainly laid on the local methods, namely on the local derivative-free local filters methods represented by the divided difference filter first order [11], [12].

As far as the statistics of the state and measurement noises are concerned, both noises are currently supposed to be zero mean with unknown covariance matrices. The noise covariance matrices can be generally estimated on-line or off-line. Due to the extensive computational demands of on-line noise covariance matrices estimation methods [13], [14], they were estimated off-line by the estimation technique based on the multi-step prediction [15].

V. EXPERIMENTS

In this section, estimation techniques described in the previous section will be used for state and parameters estimation. The goal of this section is comparison of accuracy estimation with time-variant and/or time-invariant saturation flow.

The estimation of queue lengths was tested on the synthetic micro-region. The micro-region is composed of one four-arm one-way intersection, where one is input arm and the other three are output arms, see figure 5. The micro-region is equipped with strategic and stop-line detectors on input arm and with output detectors on output arms. The data³ are measured at 90 second intervals.

The model of the traffic system, depicted in figure 5, comes from the previously introduced traffic model (1) and (2), where parameters κ_k , β_k and λ_k are unknown and time-variant. In first case, the saturation flow is assumed as time-invariant with typical value of saturation flow. In second case, the saturation flow is assumed as time-variant and it is given by relation (5). In this case, the oncoming intensity is equal to zero and so the actual saturation flow is influenced

³For the state and parameter estimation the real data, namely input and output intensities, occupancies, and cycle and green times, were used. The true queue lengths were determined by simulation software AIMSUN and they were used for the comparison with estimated lengths. The validation and calibration process of the simulator was made with respect to the particularities of the local traffic system. The validation of the queue length reconstruction was made on several types of micro-regions in Prague in accord to the guidelines specified in [16].

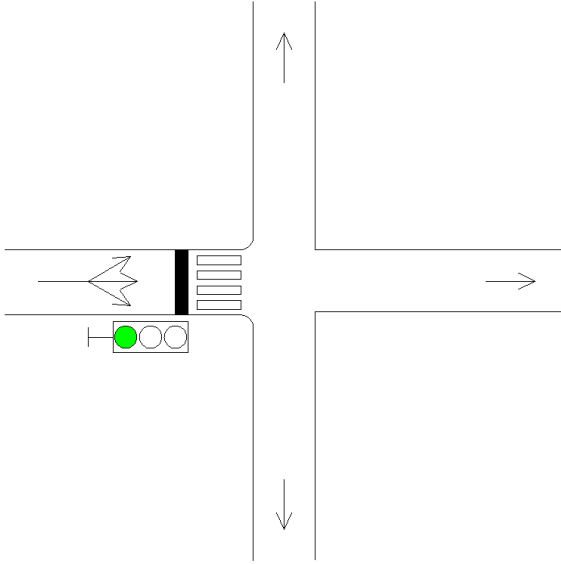


Fig. 5. Illustration of synthetic microregion.

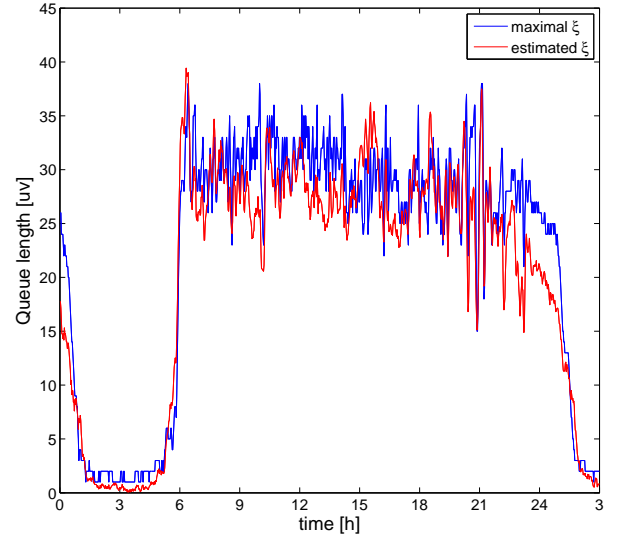
by reduction of speed because of left and/or right turning movements only. The model can be written by the following equations

$$x_{k+1} = \begin{bmatrix} \xi_{k+1} \\ I_{k+1} \\ J_{k+1} \\ O_{k+1} \end{bmatrix} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \\ x_{4,k+1} \end{bmatrix} = \begin{bmatrix} \xi_k + I_k - f(\xi_k, I_k, V_k) + w_{1,k} \\ I_k + w_{2,k} \\ J_k + w_{3,k} \\ \kappa_k \xi_k + \beta_k O_k + \lambda_k + w_{4,k} \end{bmatrix} \quad (7)$$

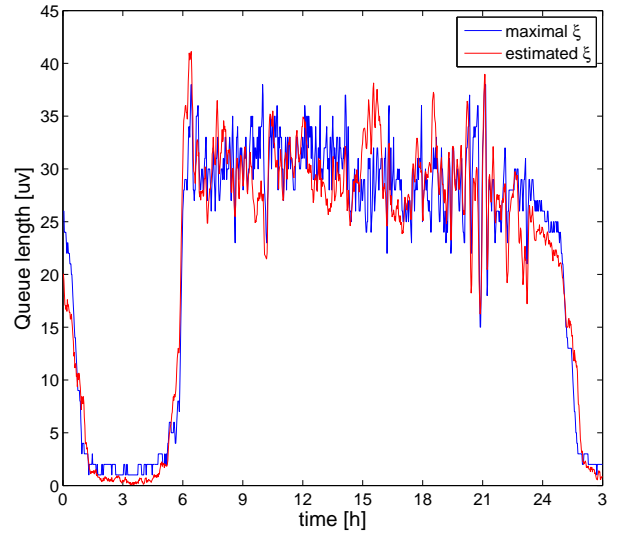
$$y_k = \begin{bmatrix} I_k \\ O_k \\ Y_{1,k} \\ Y_{2,k} \\ Y_{3,k} \\ J_k \end{bmatrix} = \begin{bmatrix} x_{2,k} + v_{1,k} \\ x_{4,k} + v_{2,k} \\ x_{3,k} + v_{3,k} \\ c_{1,k} \alpha^R x_{3,k} + v_{4,k} \\ (1 - \alpha^R - \alpha^L) x_{3,k} + v_{5,k} \\ c_{2,k} \alpha^L x_{3,k} + v_{6,k} \end{bmatrix} \quad (8)$$

The theoretical saturation flow is greater than the real because the real saturation flow takes into consideration the reduction of speed in turning, as shown in figure 6. In the night, about midnight, the green time was radically reduced. This leads to the increasing influence on traffic flow because the majority of green time is spent by starting and breaking of vehicles. In such case, the drivers are more aggressive and so the real saturation flow is subtly increased, as shown in figure 7. In this figure, it is seen how the turning movements influence the saturation flow and this approach allows to set the theoretical saturation flow with respect to the type of intersection. The real saturation flow will be recomputed with respect to the actual traffic flow behaviour (intensities and turning movements).

The estimation quality is compared on the basis of the



time-invariant saturation flow



time-variant saturation flow

Fig. 6. True and estimated queue during a whole day.

square error SE, representing average sum squared differences between the true and estimated queue lengths on input arm and one day, the root mean square error (RMSE), representing an average error of estimate in one sample period. The last item in the table I represents and average computational time required for estimation of queue lengths. It can be seen that adding time-variant saturation flow (and time-variant turning movements too) into the model causes that the estimate of the queue length is more accurate, specifically the estimation error is reduced about more than fifteen percent. The differences between the time demands are rather insignificant. Finally, note that as a state estimator the divided difference filter first order was used [11].

Finally, a few remarks will be given. Using the time-

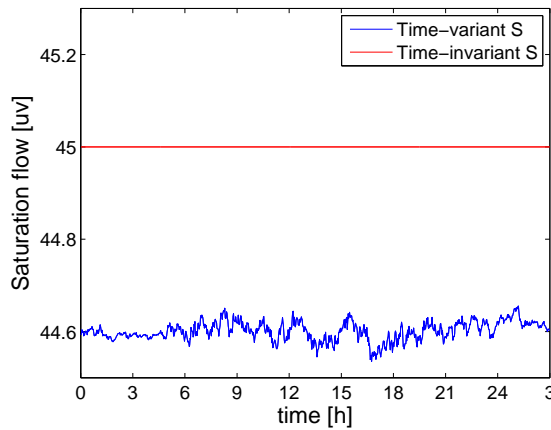


Fig. 7. Shape of time-variant and time-invariant saturation flow.

TABLE I

COMPARISON OF RESULTS GIVEN BY THE MODEL WITH TIME-VARIANT AND TIME-INVARIANT SATURATION FLOW.

	SE [uv^2]	RMSE [$uv/period$]	time [s]
Time-invariant S	$6,954 \times 10^4$	4,26	19,18
Time-variant S	$4,786 \times 10^4$	3,53	19,22

variant model of the saturation flow, which better describes the real traffic situation, leads to the superior queue length estimation quality. However, the weakness of this approach is that the time-variant saturation flow needs two input detectors, optimally the strategic and the stop-line detector. Moreover, to determine the time-variant turning movements, the output detectors are also needed. The real traffic networks are usually equipped with two input detectors and one output detectors for each line and together with acceptable computational demands of the time-variant saturation flow, it seems to be advantageous to use this approach for online queue lengths estimation.

VI. CONCLUDING REMARKS

The paper dealt with modelling of traffic system. Generally, traffic systems are usually very large scale and complex systems. Due to the complexity a lot of simplifications are introduced into a model design techniques in different modelling approaches. For example, several quantities are often modelled as time-invariant although they are evolving in the time.

In this paper, the technique for the traffic model design,

based on the traffic flow conservation principle, was introduced. Then, the assumption that the saturation flow and turning movement, as a two important traffic quantities, are time-invariant was omitted and novel relations modelling time behaviours of these quantities were presented. The novel relations are based on the assumption that the input lanes are equipped by two inductive detectors and the output lanes by one detector. Finally, the extended technique for modelling traffic system with time-variant saturation flow and turning movements were illustrated in numerical example.

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